Can we study dense matter on the lattice?

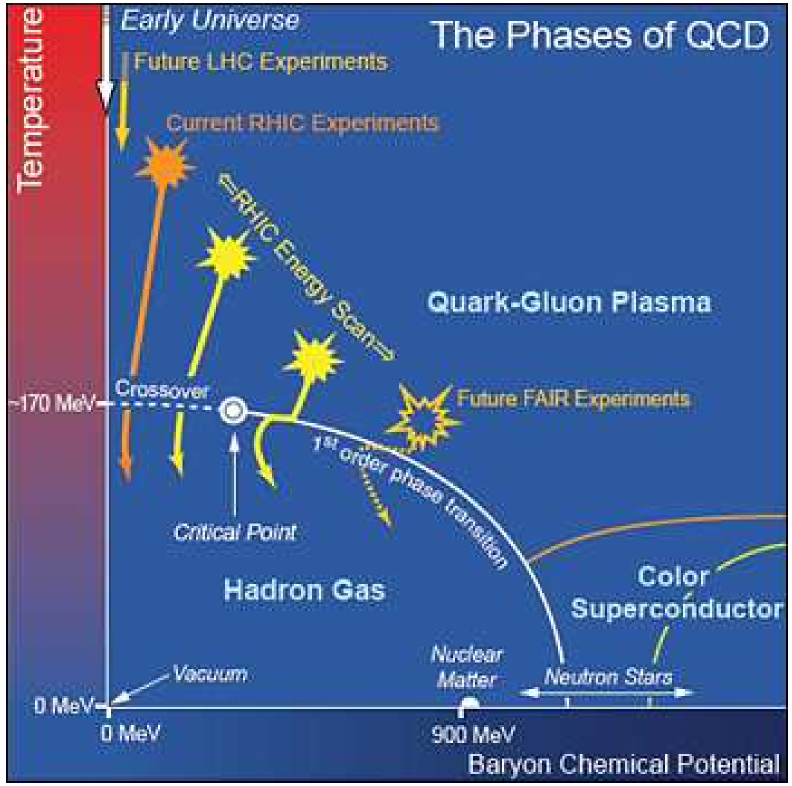
Kim Splittorff

Niels Bohr Institute / DFF - Sapere Aude

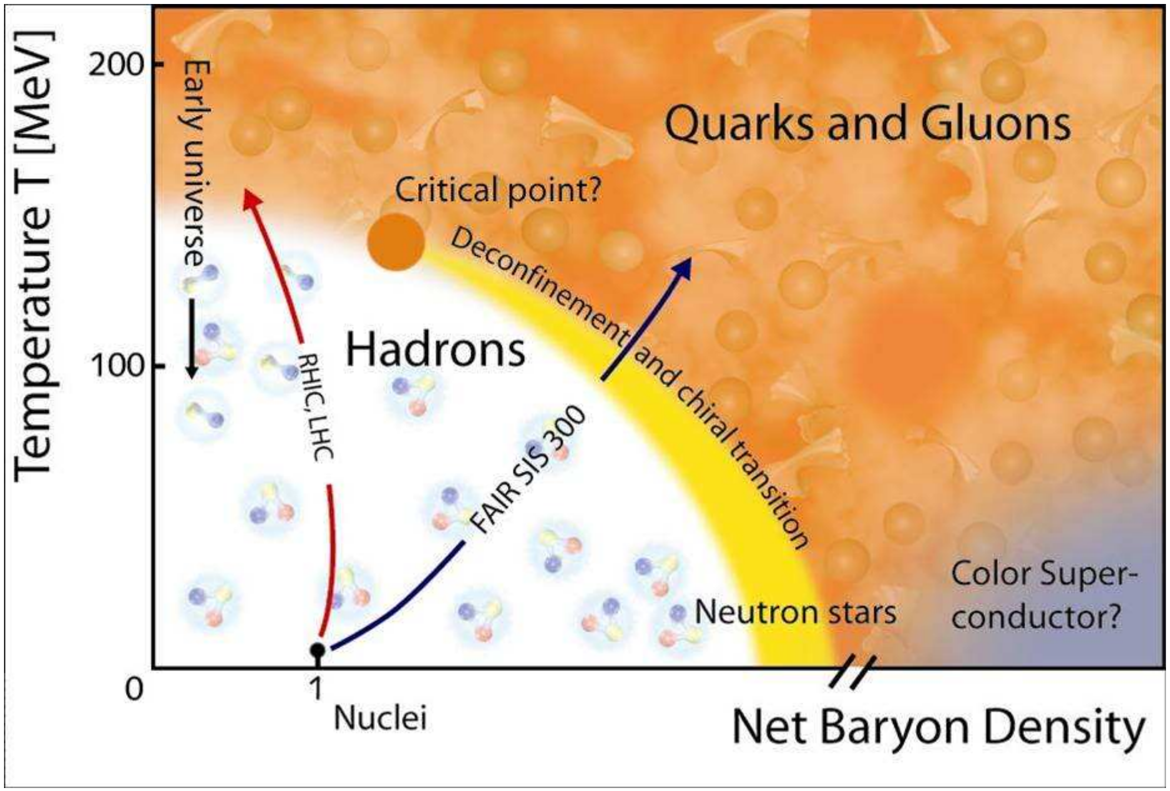
NUPECC, NBI, Copenhagen, June 14, 2012

Aim: The QCD phase diagram from first principles

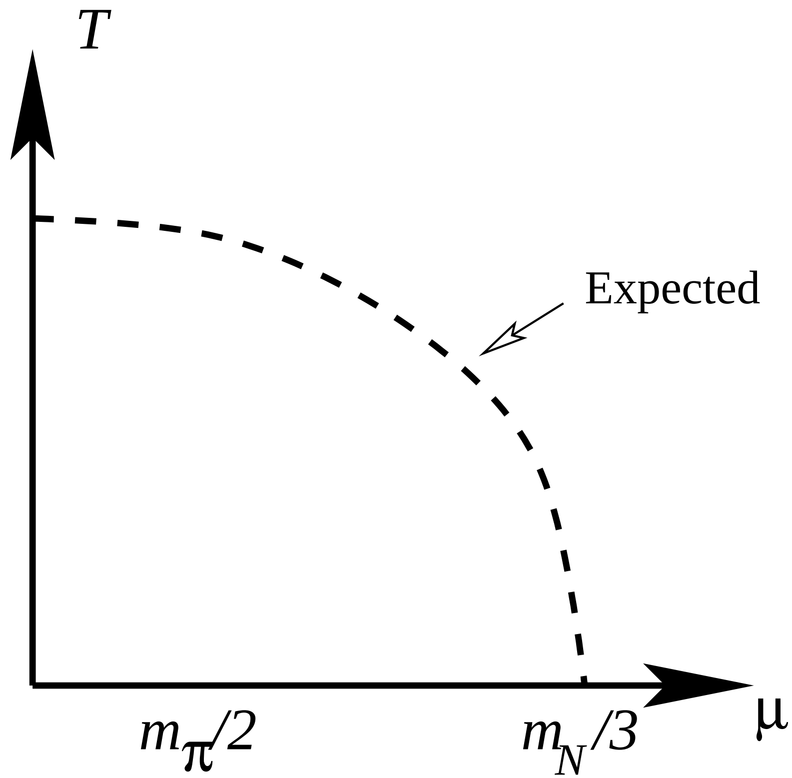
Sketch 1 RHIC



Sketch 2 GSI FAIR



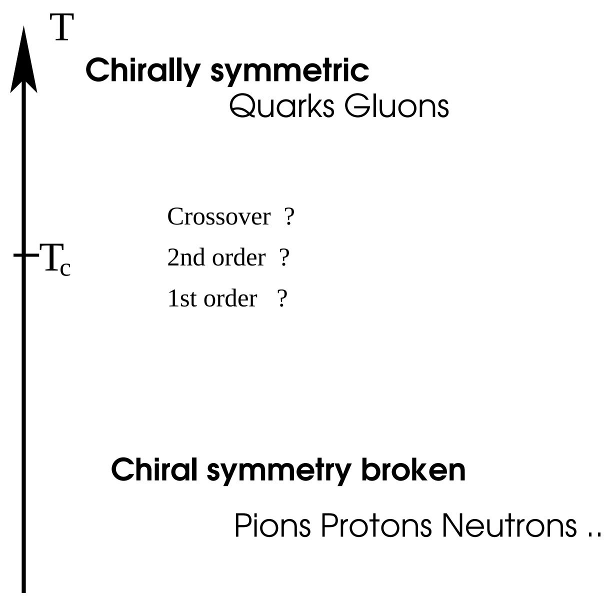
Sketch 3 What everyone agrees on



Can lattice QCD compute the QCD phase diagram ?

- **Success story** $\mu = 0$
- **How to include** μ **and the sign problem**
- $\mu \neq 0$ **status, challenges and progress**

Success: The baryon/anti-baryon symmetric world $(\mu = 0)$



Landau theory: $N_f \geq 2$ 1st order ($m_f = 0$)

Answer depends on values of m_f and $U_A(1)$ anomaly

Lattice: @ physical quark masses crossover

Pisarski, Wilczek, PRD29 (1984) 338
Aoki, Endrodi, Fodor, Katz, Szabo, Nature 443 (2006) 675

de Forcrand, Philipsen JHEP 0701 (2007) 077

Matter antimatter asymmetry ($\mu \neq 0$)

$$N > 0$$

Here: Fact which we adopt into QCD

Grand canonical approach: *Fix μ determine N*

$$N = \frac{1}{V} \partial_\mu \log Z(\mu)$$

How to include μ in Z

μ is conjugate variable to N

$$\mu N = \mu \langle q^\dagger q \rangle = \mu \langle \bar{q} \gamma_0 q \rangle$$

$$\mathcal{L}_{\mathrm{QCD}} = \bar{q}(D_\eta \gamma_\eta + \mu \gamma_0 + m) q + \mathrm{Gluons}$$

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μ *enters as the 0th component of the gauge field*

$$\mathcal{L}_{\mathrm{Lattice\ QCD}} = ... + e^{a\mu}\bar{q}_x\gamma_0 U_{x,x+\hat{0}} q_{x+\hat{0}} + e^{-a\mu}\bar{q}_{x+\hat{0}}\gamma_0 U^\dagger_{x,x+\hat{0}} q_x + ...$$

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Works fine for free quarks

Hasenfratz, Karsch, PLB 125 (1983) 308

The sign problem

$$Z_{N_f=2} = \int dA \, \det{}^2(D + \mu\gamma_0 + m) \, e^{-S_{\rm YM}}$$

Anti Hermitian

Hermitian

$$\det{}^2(D + \mu\gamma_0 + m) = |\det(D + \mu\gamma_0 + m)|^2 e^{2i\theta}$$

The measure is not real and positive

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No Monte Carlo sampling of A_η *at* $\mu \neq 0$

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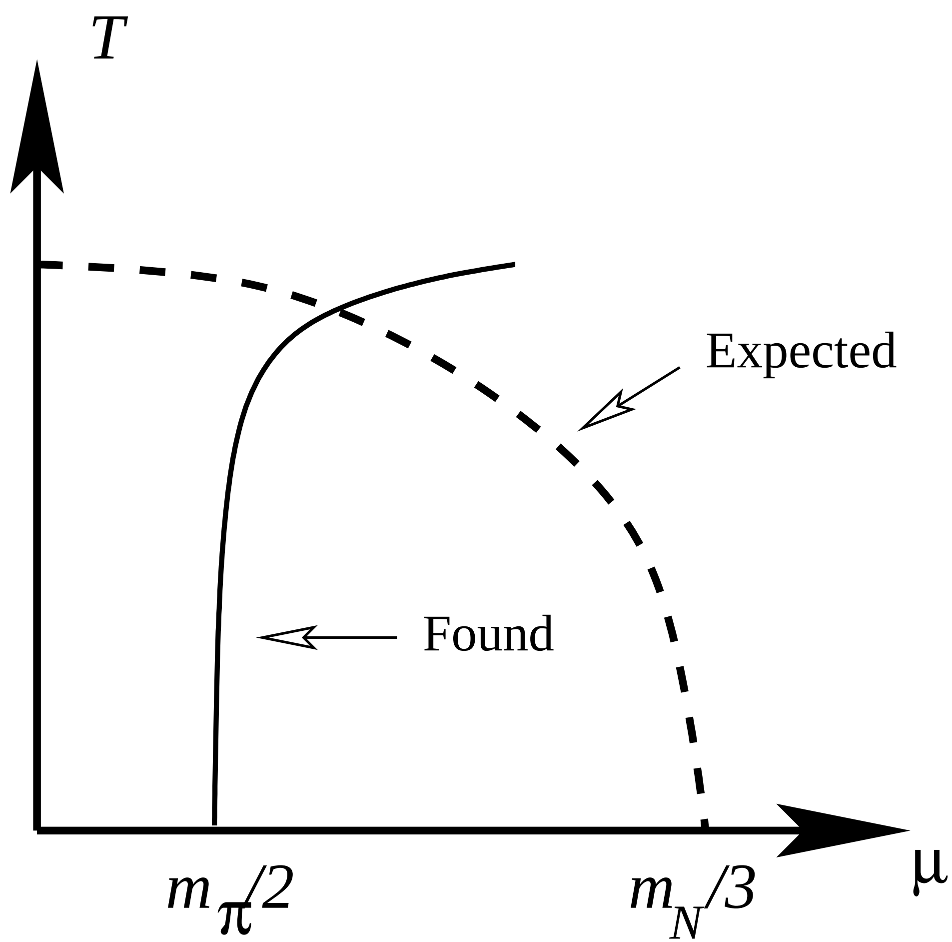
What if we simply ignore $e^{2i\theta}$?

If we ignore $e^{2i\theta}$: *phase quenched QCD*



Kogut, Sinclair Phys.Rev.D77:114503,2008

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$$|\det(D + \mu\gamma_0 + m)|^2 = \det(D + \mu\gamma_0 + m)\det(D - \mu\gamma_0 + m)$$

ISOSPIN CHEMICAL POTENTIAL

Alford Kapustin Wilczek PRD 59 (1999) 054502

The QCD Phase Diagram at nonzero isospin chemical potential

The pions have nonzero isospin $\Rightarrow$ BEC of pions

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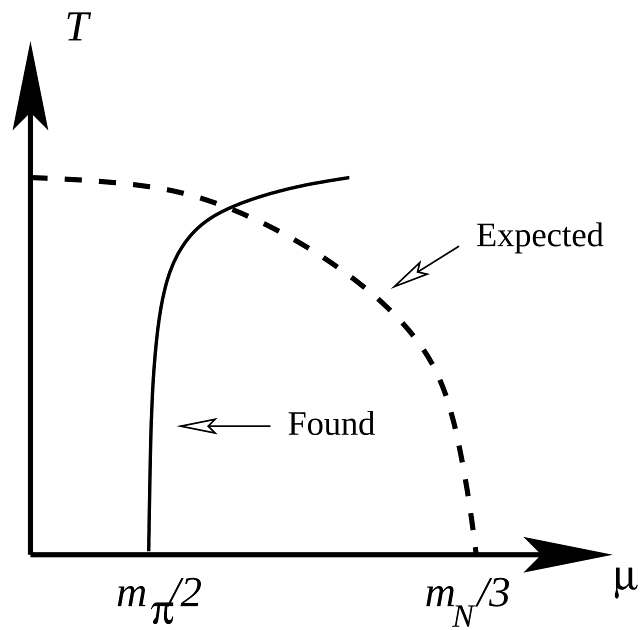

The QCD Phase Diagram at nonzero isospin chemical potential

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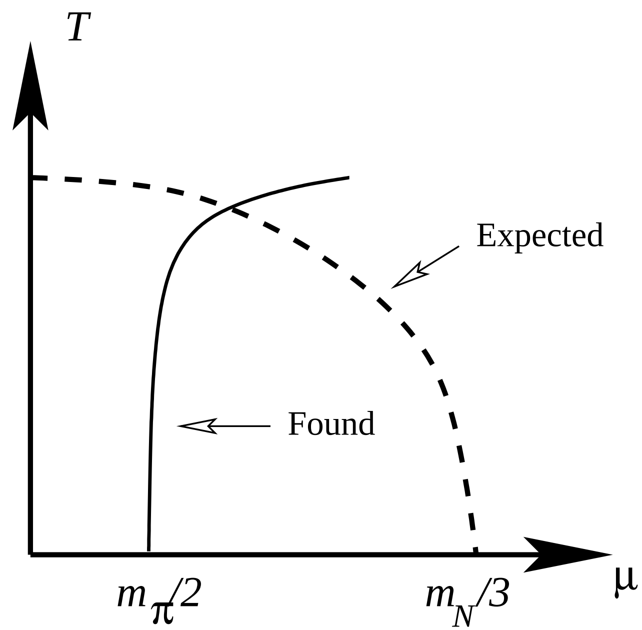


Conclude: The phase $e^{2i\theta}$ is *highly* relevant for the phase diagram

Son, Stephanov Phys.Rev.Lett. 86 (2001) 592

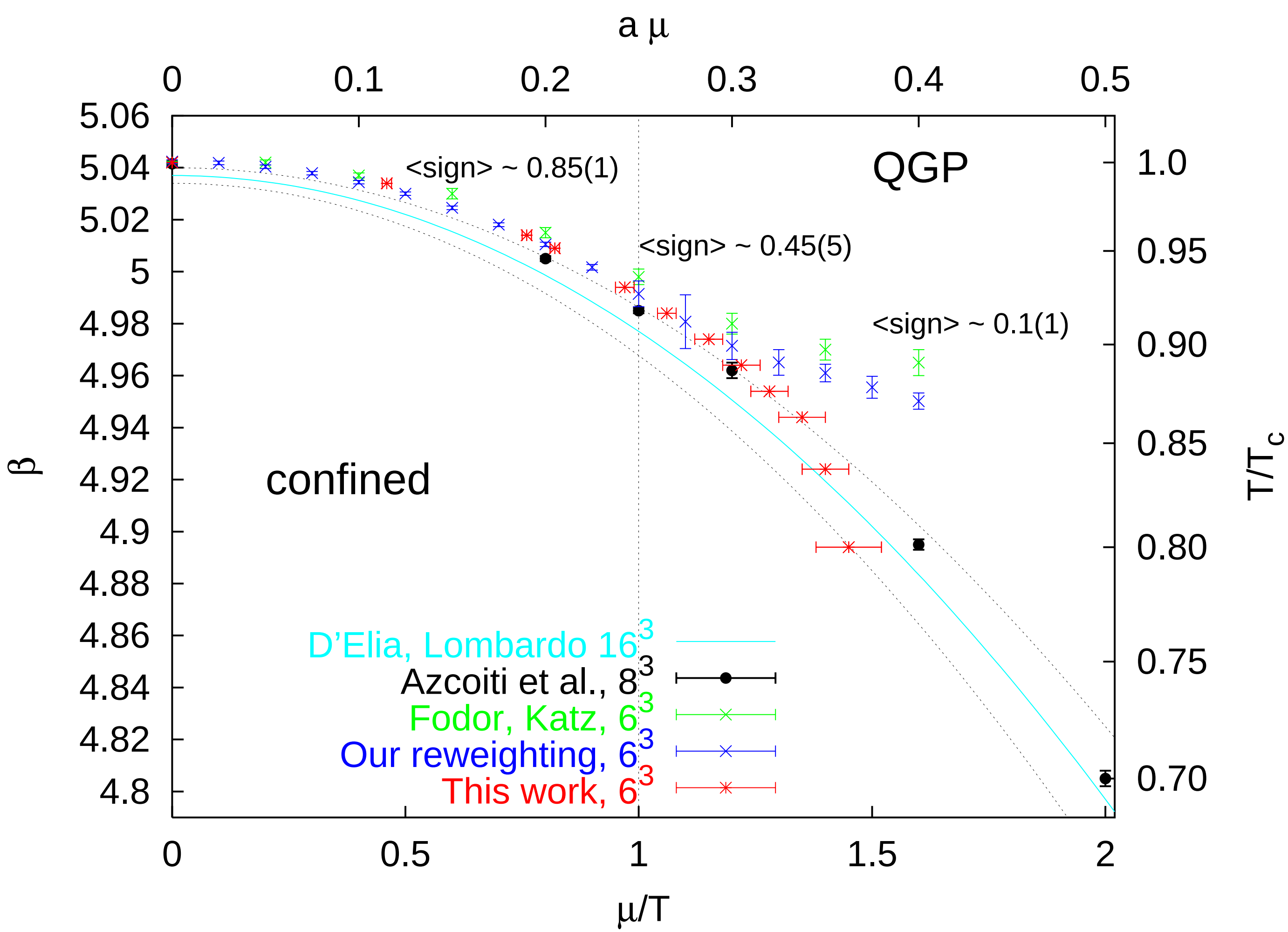
Kogut, Sinclair Phys.Rev.D77:114503,2008

So include the phase factor in Lattice QCD

Method	Idea	Challenge
Reweighting	Absorb the sign in the observable	Exponential cancellations
Taylor expansion	Expand at $\mu = 0$	Higher order terms
Imaginary μ	Determine the analytic function	Control the extrapolation
Density of states	Use the distribution of the phase	Determine the distribution
Canonical ensemble	Work at fixed baryon number	Fix the baryon number

review Lombardo, Splittorff, Verbaarschot arXiv:0912.4410

The curvature and the average sign

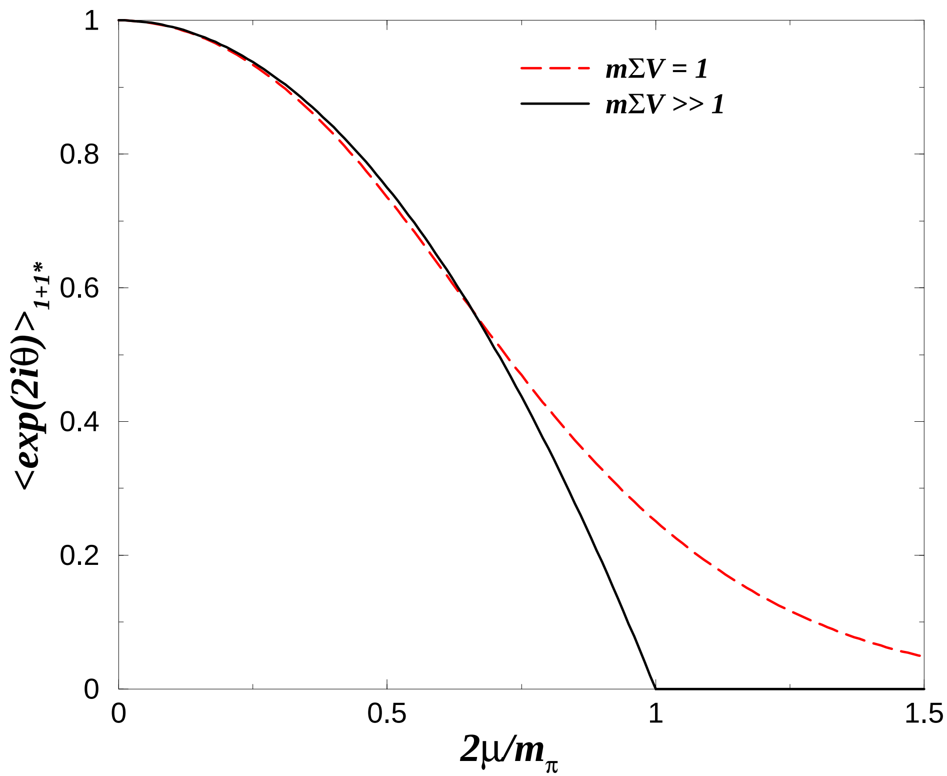


de Forcrand Philipsen JHEP PoS LAT2005 (2005) 016

Progress: Karsch B-J Schaefer Wagner Wambach PoS(Lattice 2011)219

The average phase factor

$$\langle e^{2i\theta} \rangle_{1+1^*} = \frac{I_0(\hat{m})^2 - I_1(\hat{m})^2}{2e^{2\hat{\mu}^2} \int_0^1 dt t e^{-2\hat{\mu}^2 t^2} I_0(\hat{m}t)^2}$$



Splittorff Verbaarschot PRL 98 (2007) 031601

Particular challenging for $\mu > m_\pi/2$

1) $\langle e^{2i\theta}\rangle \sim e^{-V(1-m_\pi^2/4\mu^2)^2\mu^2 F_\pi^2}$

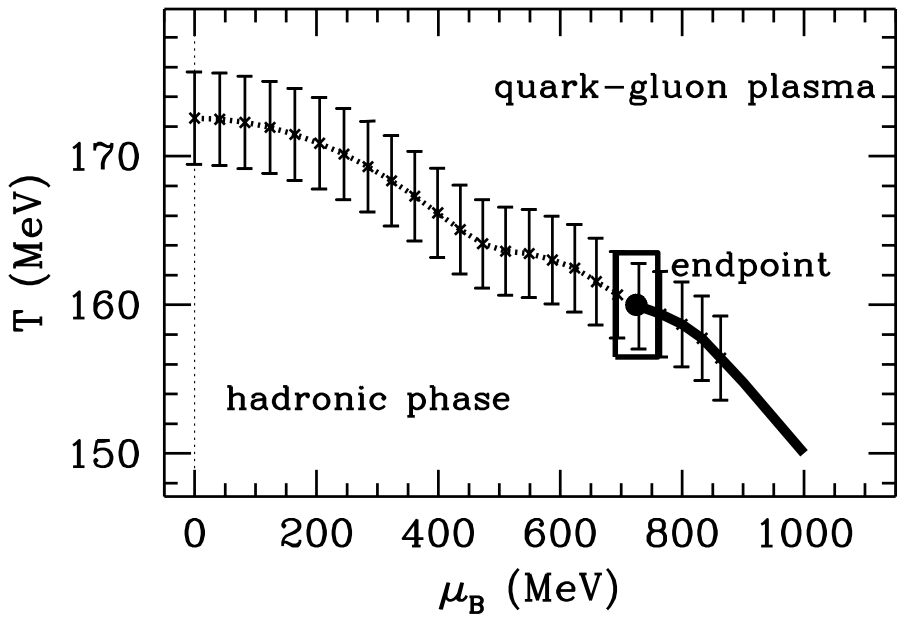
2) New Banks Casher relation (essential for SBχS)

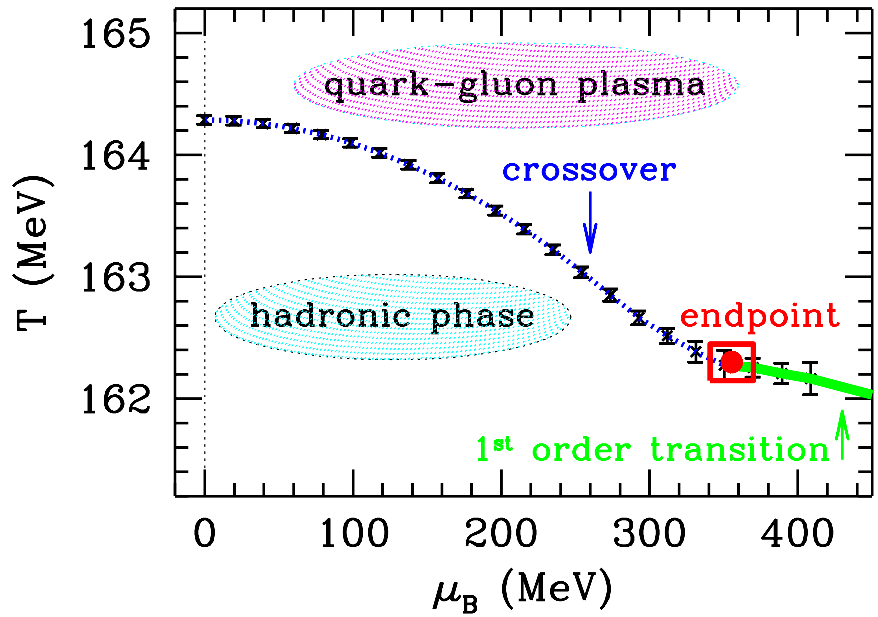
3) $\bar\psi\psi$ and n_B fluctuate wildly (Lorentzian dist)

reviews Splittorff Verbaarschot arXiv:0809.4503

Lombardo, Splittorff, Verbaarschot arXiv:0912.4410

Lattice measurement of endpoint (Reweighting)

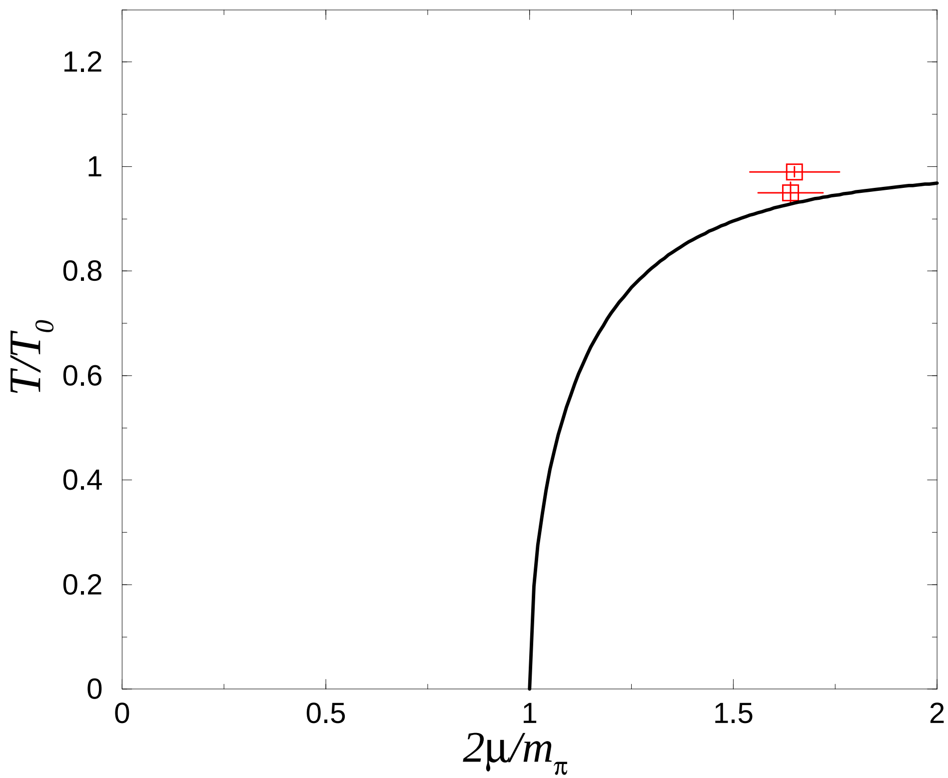




Fodor Katz JHEP 0203:014,2002; JHEP 0404:050,2004

Lattice measurement of endpoint (Rewieghting)

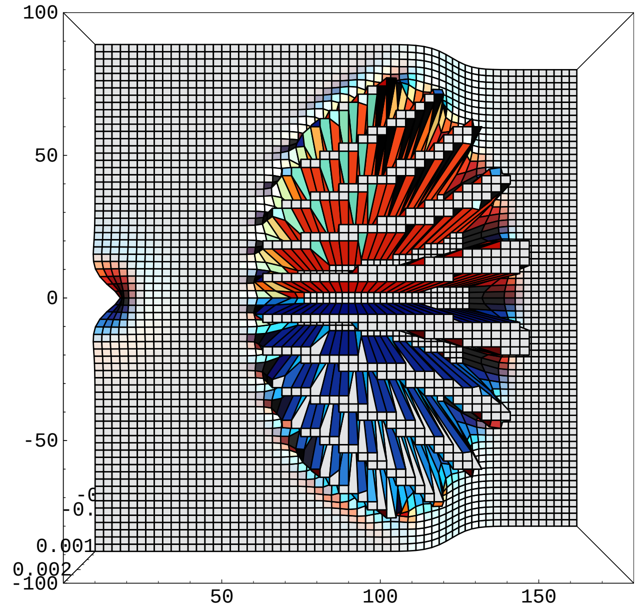
- Scale the axis by T_C and $m_\pi/2$



Fodor Katz JHEP 0203:014,2002; JHEP 0404:050,2004
Splittorff, hep-lat/0505001, PoS LAT2006:023,2006

Philipsen 0710.1217

For $\mu > m_\pi/2$: Oscillations Period $\sim 1/V$ Amp $\sim e^{+V}$



In: Chiral Dirac eigenvalue density, phase distribution, baryon number distribution, baryon Dirac eigenvalue density, ...

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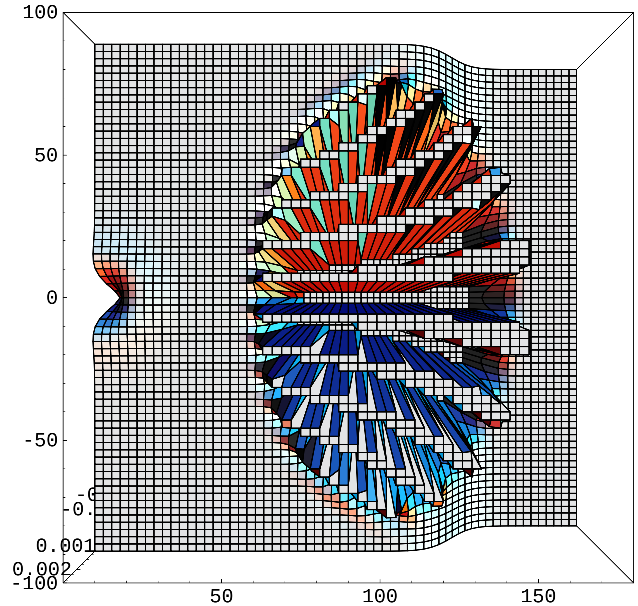

In: Chiral Dirac eigenvalue density, phase distribution, baryon number distribution, baryon Dirac eigenvalue density, ...

but only on this scale!

Akemann Osborn Splittorff Verbaarschot NPB 712 (2005) 287

Ipsen Splittorff arXiv:1205.3093

Progress: Complex Langevin

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Method	Idea	Challenge
Complex Langevin	Stochastic flow in complex plane	Optimization

Problems for CL are 'orthogonal' to $\langle e^{2i\theta}\rangle \ll 1$

Can resolve oscillations with period $1/V$ and amp $\exp(V)$

Parisi, Phys. Lett. 131 B (1983) 393
Aarts, Splittorff, JHEP 1008:017,2010

Aarts, Seiler, Stamatescu Phys.Rev.D81:054508,2010

Progress: Subset method

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Method	**Idea**	**Challenge**
Subset method	Sum over subset of configs	Apply it to QCD

So far solves the $\exp(V)$ hard sign problem in Random Matrix Theory

Bloch, PRL.107:132002,2011

Bloch, Bruckmann, Gattringer, Splittorff, *in progress*

Conclusions

Interplay between lattice QCD and analytic QCD has

1) allowed us to understand the sign problem

2) understand lattice results

3) solid progress towards a solution

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