Roman Bertle, Manfried Faber, Roman Höllwieser

Center Vortices, Confinement and Chiral Symmetry Breaking

Michael Engelhardt, Jeff Greensite,Urs Heller,
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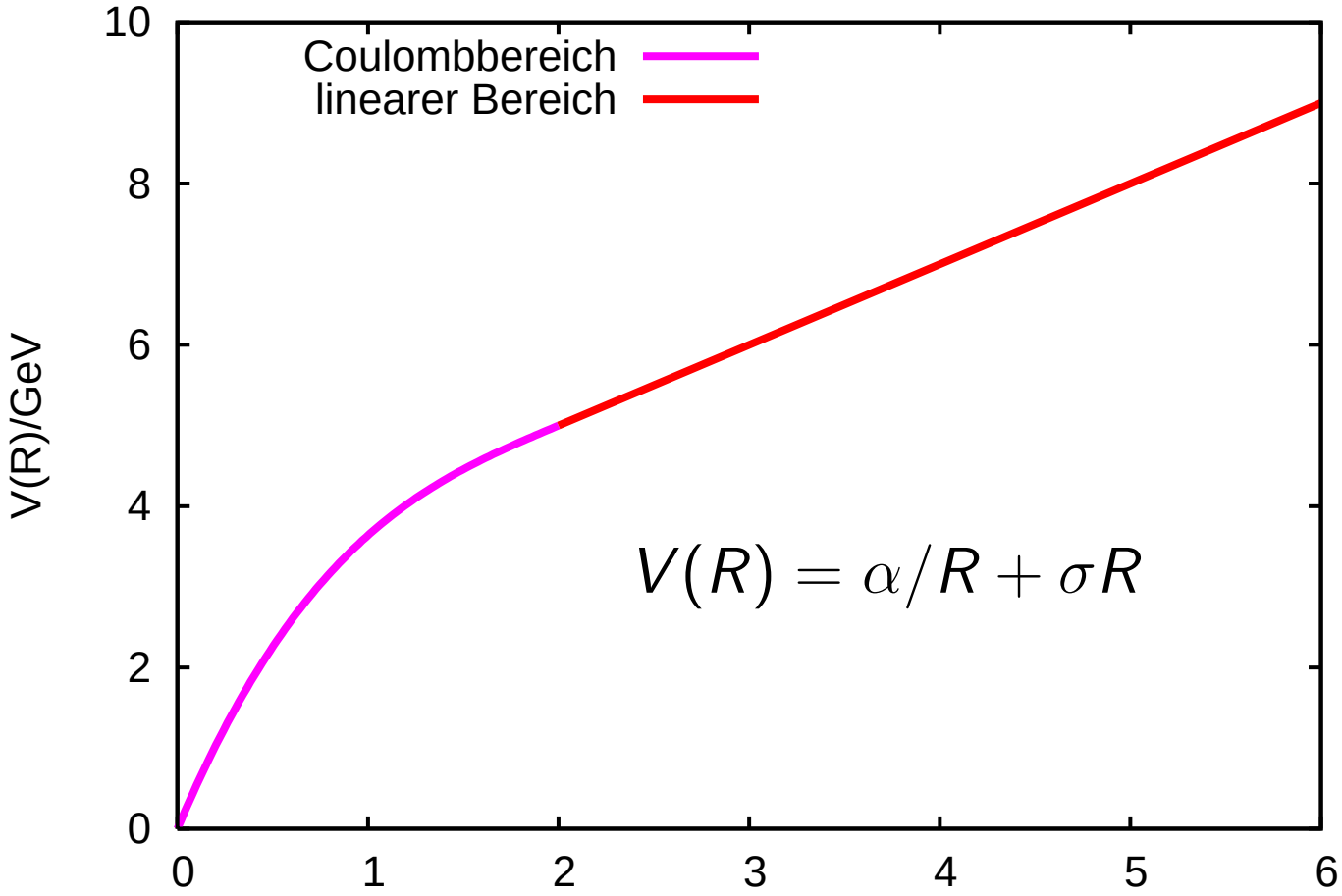
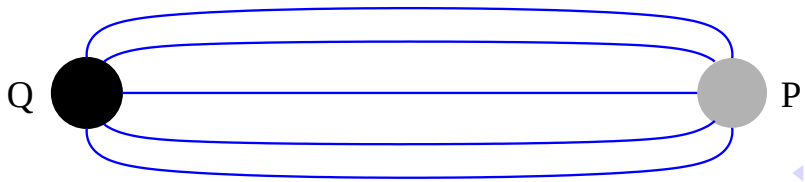
Bratislava, New York, San Francisco, Tübingen, Vienna

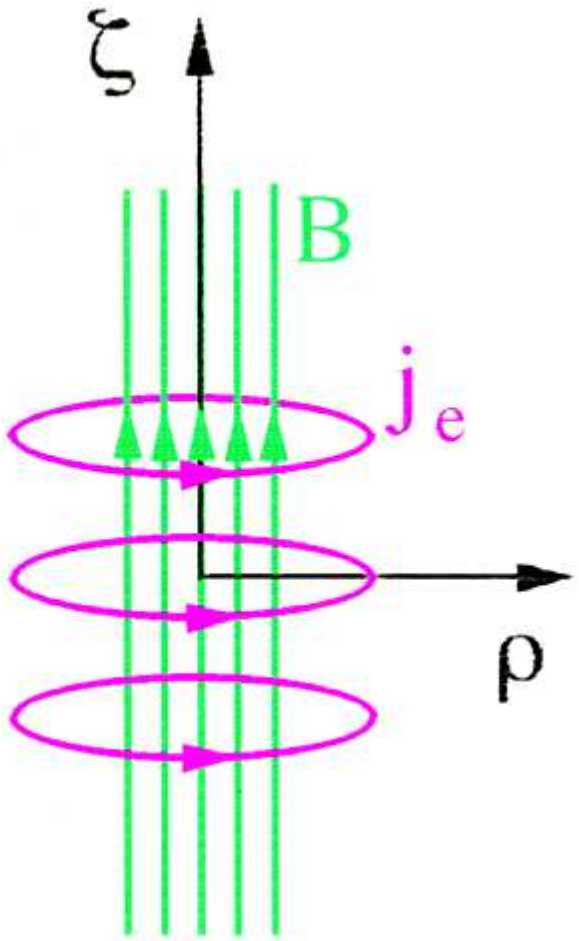
Quark Confinement





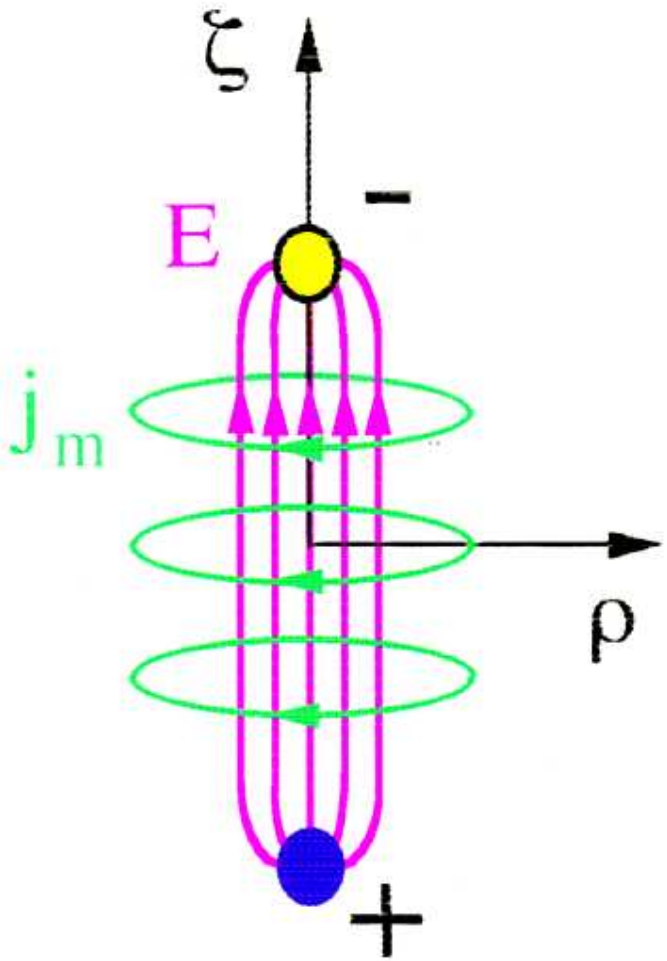
Confinement due to Magnetic Monopoles

type II superconductor



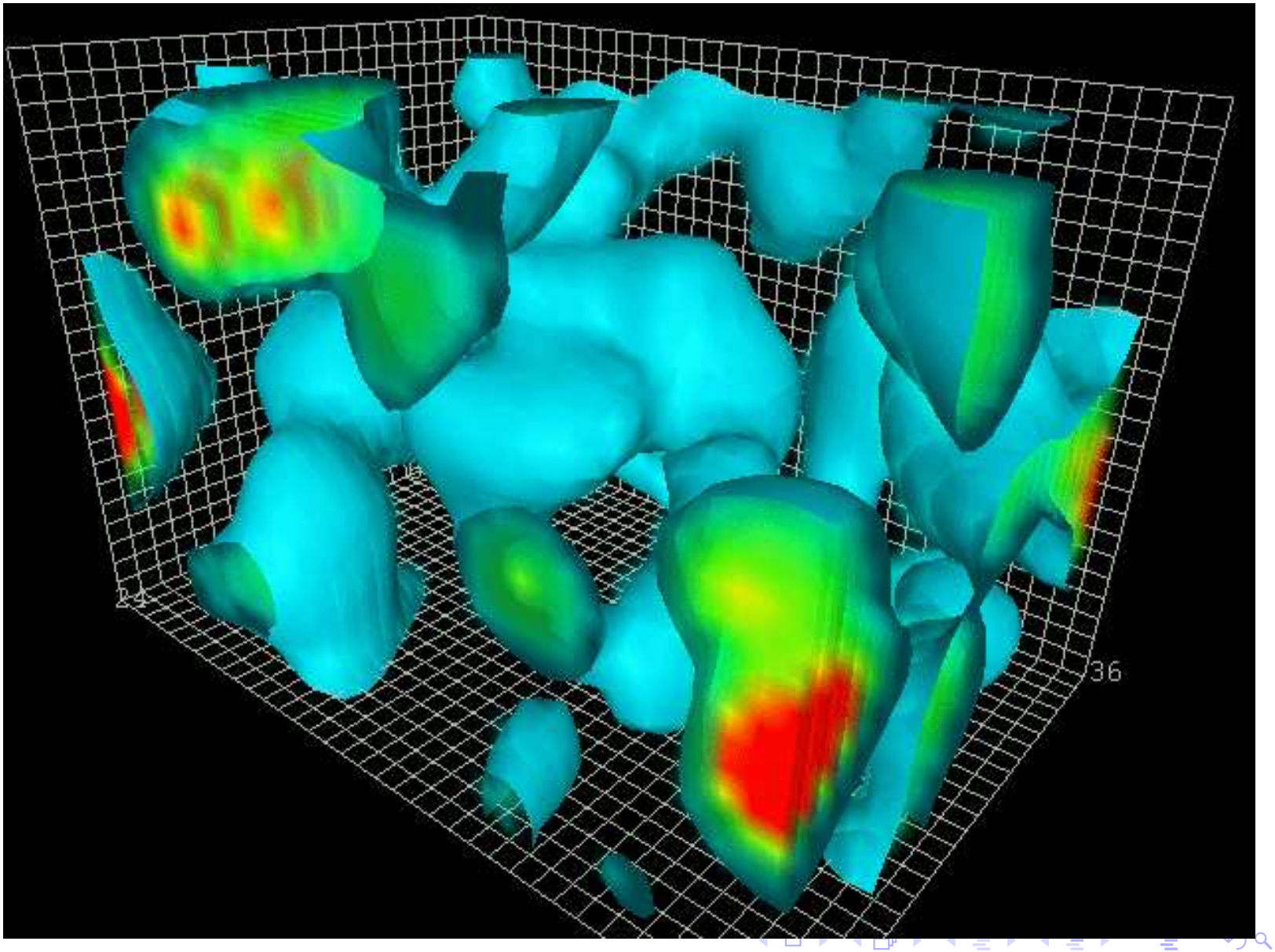
magnetic fluxoid quantisation

dual superconductor



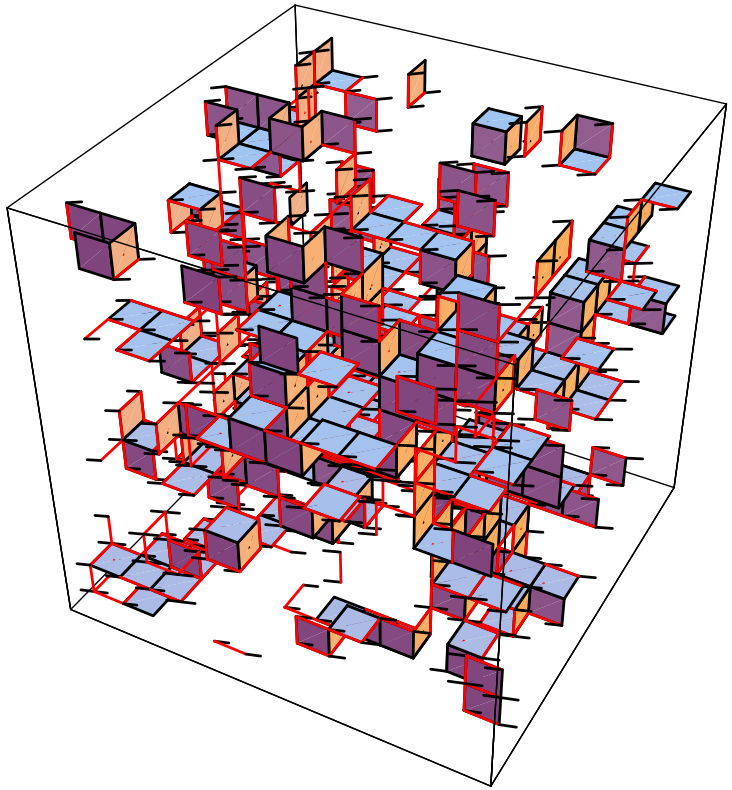
electric fluxoid quantisation

QCD Vacuum Action Density



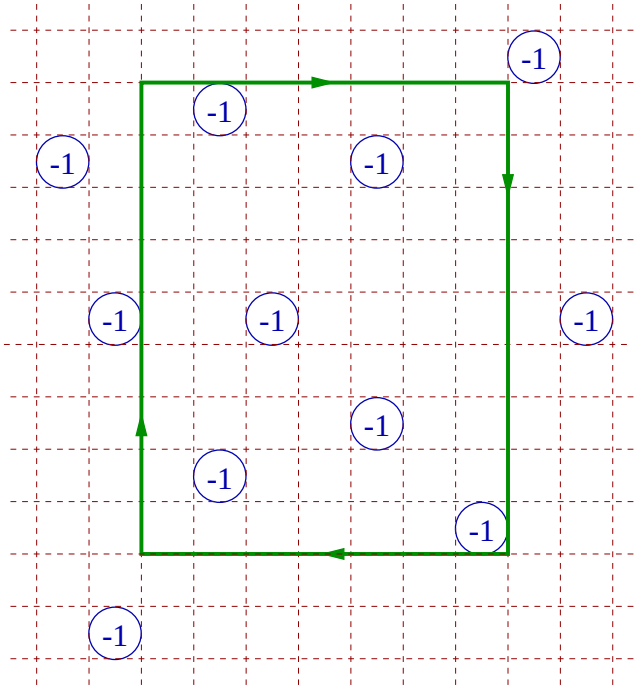
Structure of P-Vortices

In 4D they form closed 2D-surfaces in Dual Space, Random Structure



3-dimensional cut through the dual of a 12^4-lattice.

Area law for center-projected Wilson loops



denote f the probability that a plaquette has the value *-1*

$$\langle W(A)\rangle = [f(-1) + (1-f)\cdot 1]^A = \exp[\underbrace{\ln(1-2f)}_{-\sigma} A], =$$

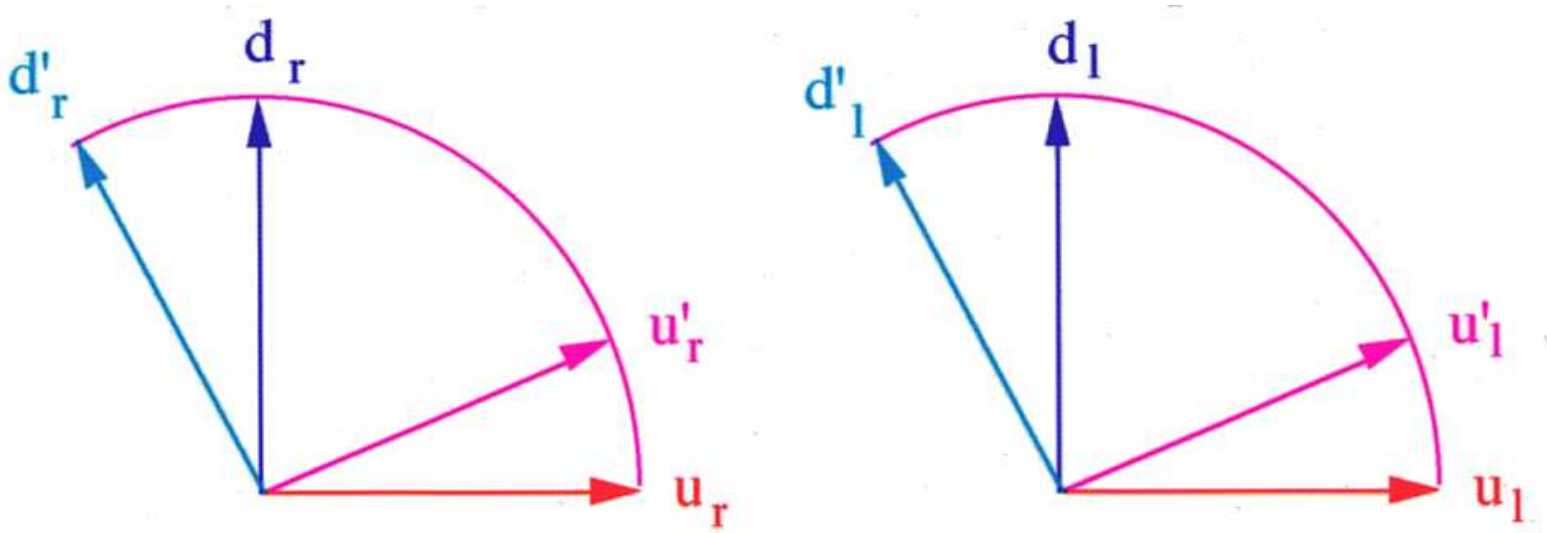
$$= \exp[-\sigma R \times T], \qquad \sigma \equiv -\ln(1-2f) \approx 2f$$

Chiral Symmetry Breaking

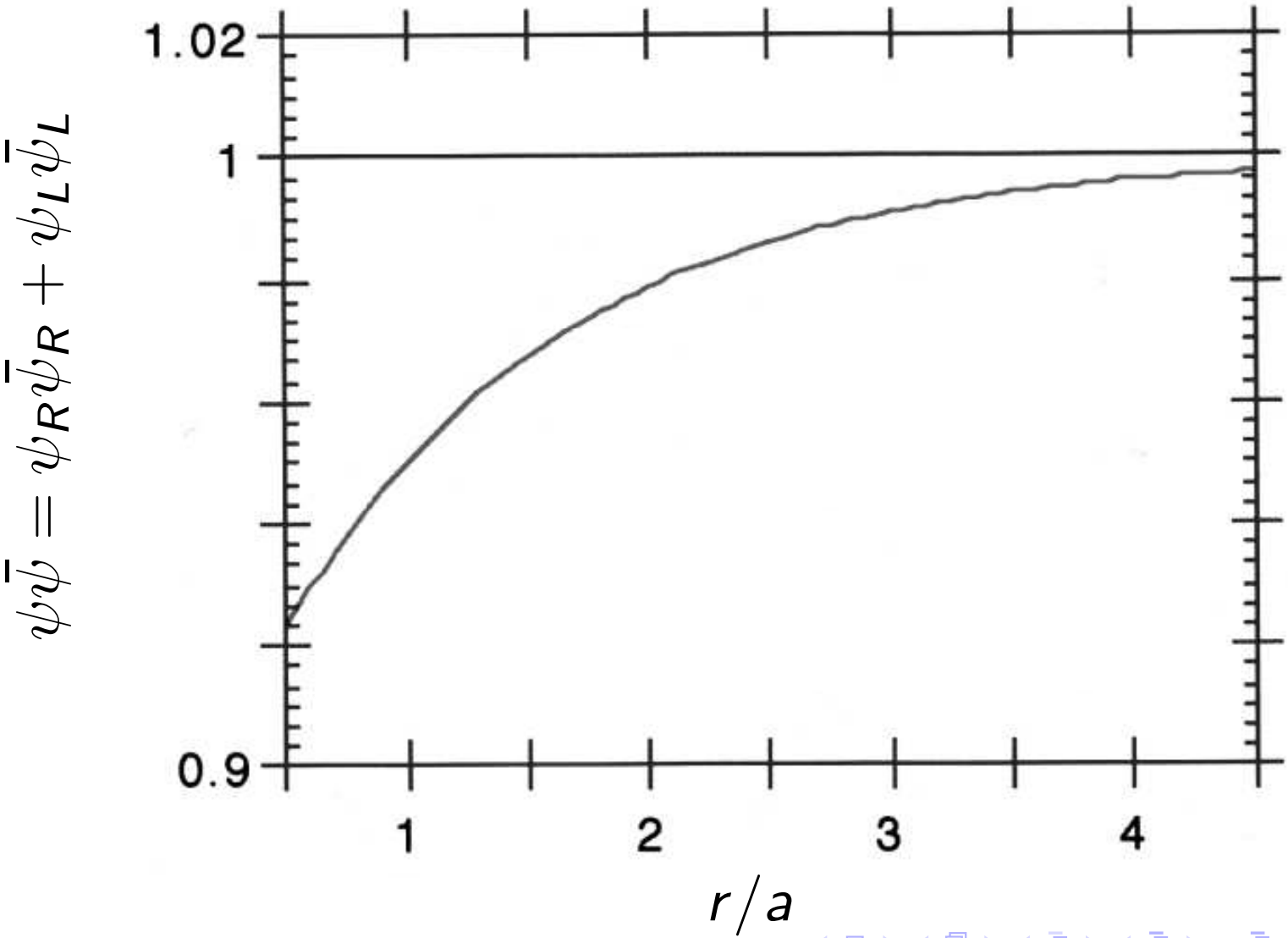
$$m_q + m_{\bar{q}} = 620\text{MeV}, \qquad m_\pi = 140\text{MeV}$$

$\Rightarrow$ pions near Goldstone bosons of Chiral Symmetry Breaking

QCD dynamics couple quark flavors



Chiral Symmetry Breaking



Banks-Casher relation

Chiral symmetry breaking $\Longrightarrow$

$\Longrightarrow$ Low-lying eigenmodes of Dirac operator

$$\bar{\psi}\psi = -\lim_{m\longrightarrow 0}\lim_{V\longrightarrow\infty}\left\langle \frac{1}{V}\sum_n \frac{1}{\mathrm{i}\lambda_n + m}\right\rangle$$

Non-zero eigenvalues appear in pairs $\pm\mathrm{i}\lambda_n$

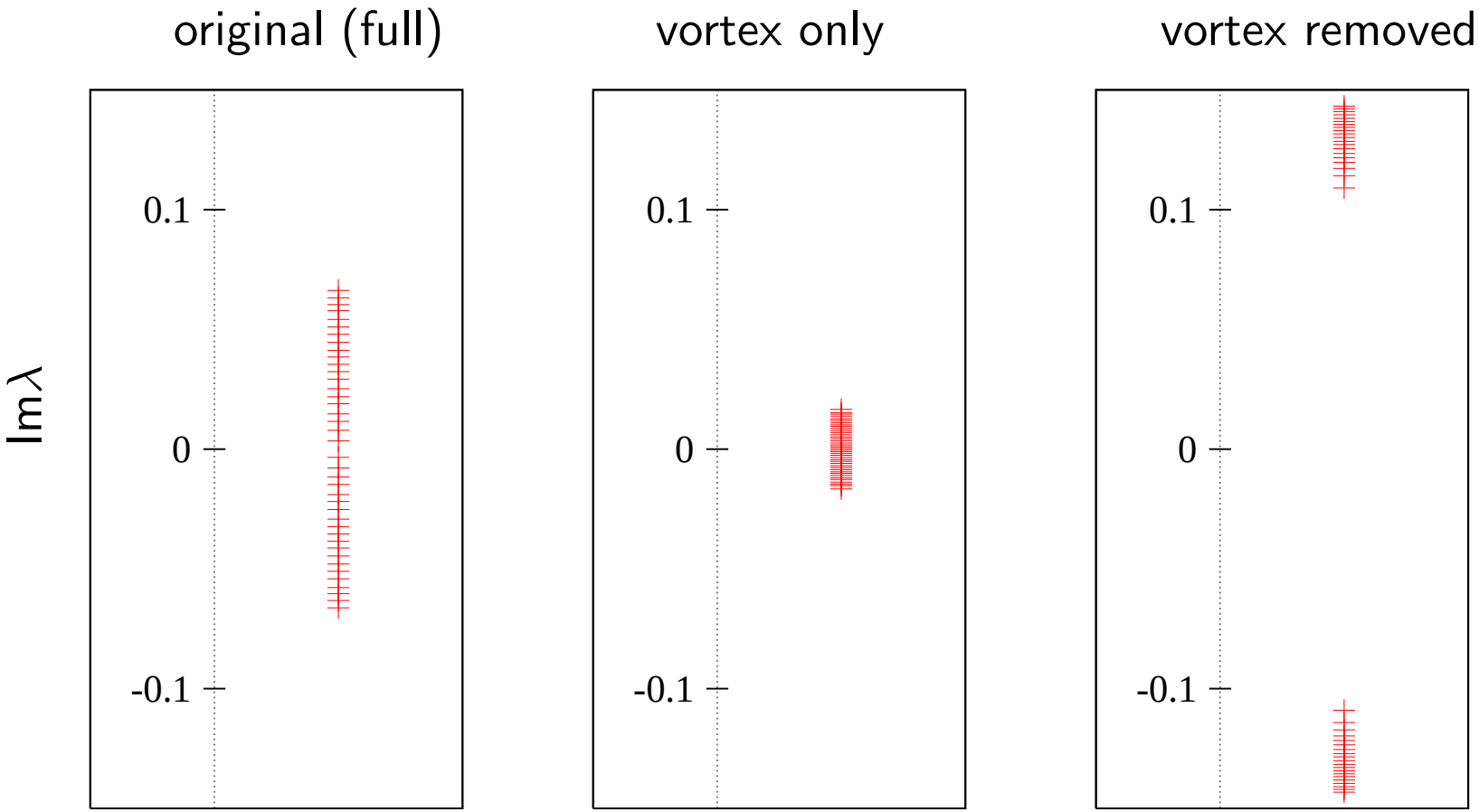
$$\lim_{m\longrightarrow 0}\frac{2m}{\lambda_n^2 + m^2} \longrightarrow \pi\delta(0)$$

Chiral condensate $\Longrightarrow$ Density of Near-Zero modes.

$$\bar{\psi}\psi = \frac{\pi\rho(0)}{V}$$

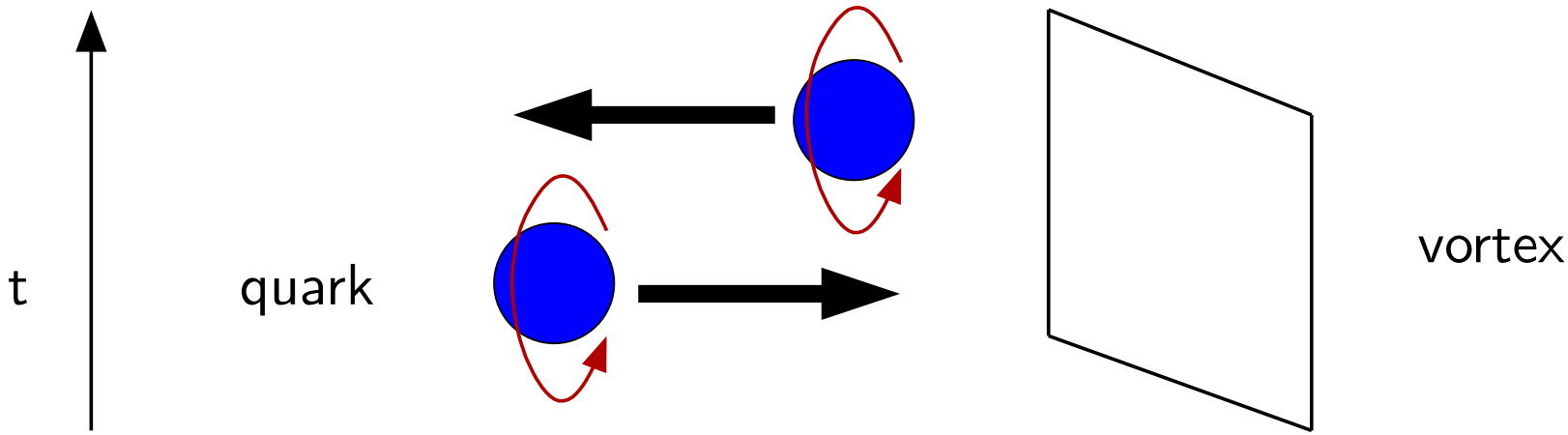
➜ *Banks, Casher, 1980*

Eigenvalues of the Staggered Fermion operator



Conclusions

QCD Dynamics break Chiral Symmetry



Strong indications that Center Vortices explain Confinement and Chiral Symmetry Breaking.

Energy Level Displacements of Kaonic Deuterium in the Excited $2p$ State and Low–Energy $\bar{K}N$ and K^-d Scattering

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March 13, 2009 / Vienna

Energy Level Displacement of Kaonic Hydrogen in the Ground State by Strong $\bar{K}N$ Low-Energy Interactions

- M. Iwasaki *et al.* (the KEK Collaboration), Phys. Rev. Lett. **78**, 3067 (1997); T. M. Ito *et al.* (the KEK Collaboration), Phys. Rev. C **58**, 2366 (1998)

$$-\epsilon_{1s}^{(\mathrm{exp})} + i\,\frac{\Gamma_{1s}^{(\mathrm{exp})}}{2} = (-323 \pm 63 \pm 11) + i\,(204 \pm 108 \pm 50)\,\mathrm{eV}$$

- G. Beer *et al.* (the DEAR Collaboration), Phys. Rev. Lett. **94**, 212302 (2005)

$$-\epsilon_{1s}^{(\mathrm{exp})} + i\,\frac{\Gamma_{1s}^{(\mathrm{exp})}}{2} = (-193 \pm 37 \pm 6) + i\,(125 \pm 56 \pm 15)\,\mathrm{eV}$$

Complex S–Wave K^-p Scattering Length

Deser-Goldberger-Baumann-Thirring-Trueman formula

$$-\epsilon_{1s} + i\,\frac{\Gamma_{1s}}{2} = \frac{2\pi}{\mu}\,\tilde{a}_{K^-p}\,|\Psi_{1s}(0)|^2 = 2\,\alpha^3\mu^2\,\tilde{a}_{K^-p}$$

$$f(k)_{K^-p} = \frac{1}{2ik}\left(\eta(k)\,e^{i2\delta(k)} - 1\right)\Big|_{k=0} = \tilde{a}_{K^-p} = \mathrm{Re}\,\tilde{a}_{K^-p} + i\,\mathrm{Im}\,\tilde{a}_{K^-p}$$

Experimental data

$$\tilde{a}^{KEK}_{K^-p} = (-0.784 \pm 0.153 \pm 0.027) + i\,(0.494 \pm 0.252 \pm 0.121)\ \mathrm{fm}$$

$$\tilde{a}^{DEAR}_{K^-p} = (-0.468 \pm 0.090 \pm 0.015) + i\,(0.302 \pm 0.135 \pm 0.036)\ \mathrm{fm}$$

Measurement of Energy Level Displacements

Analysis of X–ray from $(K^- p)_{2p} \to (K^- p)_{1s} + \gamma$ transitions

$$n \simeq \sqrt{\frac{\mu_K}{m_e}} = 25 \quad \mu_K = \frac{m_K m_p}{m_K + m_p} = 323.5\,\mathrm{MeV}$$

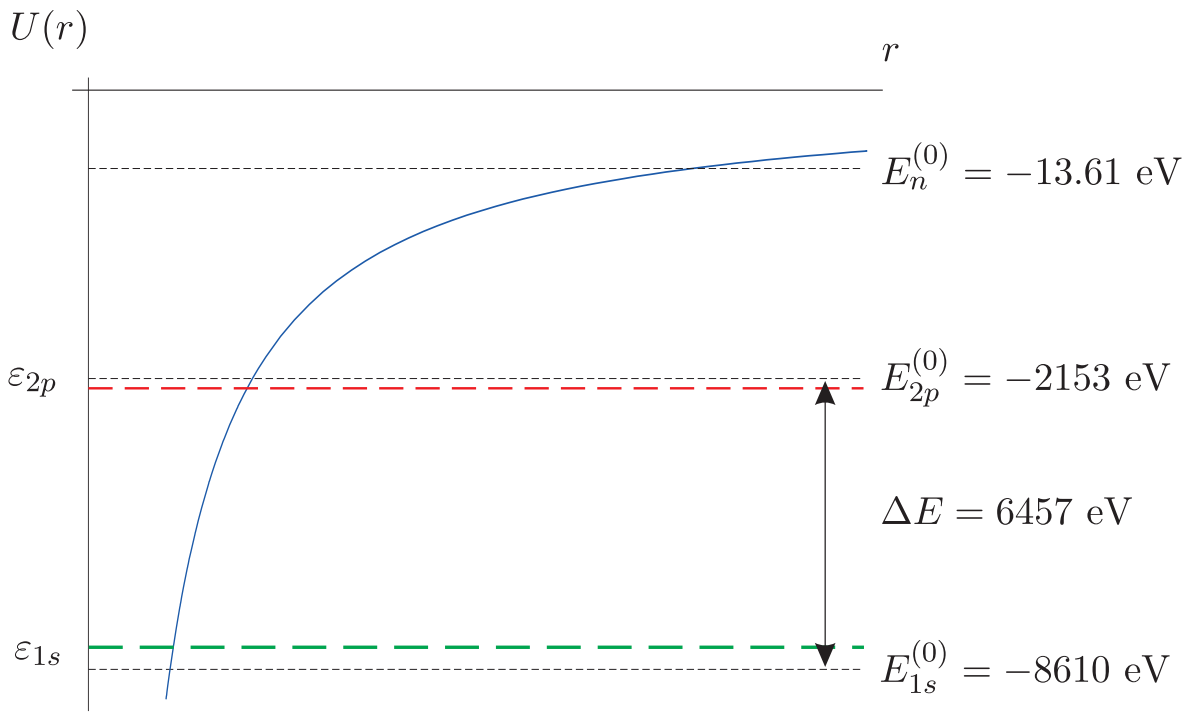
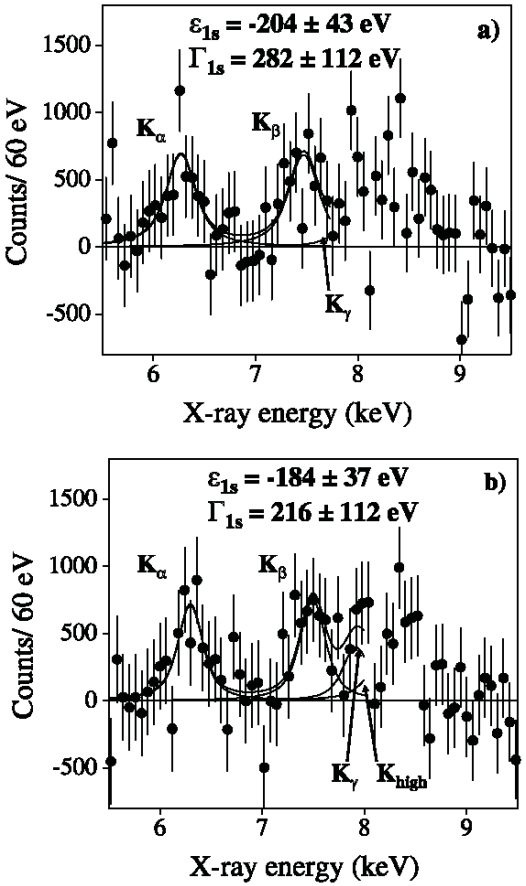
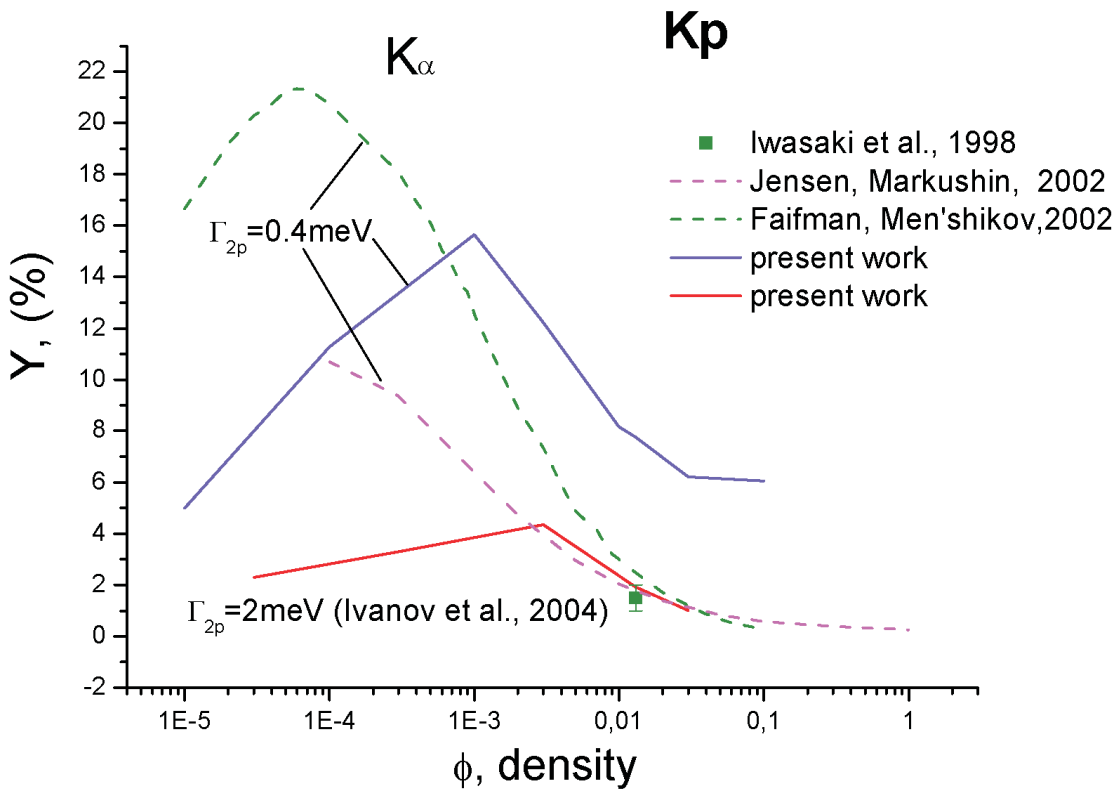


Figure: Cascade transitions of kaonic hydrogen $(K^- p)_n \to (K^- p)_{2p}$.

Experimental X–Ray Spectra

Mark Faifman (EXA2005) : $Y^{\mathrm{exp}} = (1.1 \pm 0.3)\,\%$





T. Koike *et al.*, Phys. Rev. C **53**, 79 (1996): $\Gamma_{2p} > 1\,meV$

A.N.I. *et al.*, Phys. Rev. A **71**, 052508 (2005): $\Gamma_{2p} = 2\,meV$

S–wave Scattering Length of Kaonic Deuterium

$$-\epsilon_{1s} + i\frac{\Gamma_{1s}}{2} = 2\alpha^3\mu^2\,\tilde{a}_{K^-d}$$

$$\rightarrow \tilde{a}_{K^-d} = \frac{m_d}{m_K + m_d}\int d^3x\,|\Phi_d(\vec{r})|^2\,\hat{A}_{K^-d}(r)$$

Faddeev equation: $T_{K^-d} = T_p + T_n$

S. S. Kamalov *et al.*, Nucl. Phys. A **690**, 494 (2001)

$$\begin{aligned} T_p &= t_p + t_p G_0 T_n + t_p^x G_0 T_n^x \\ T_n &= t_n + t_n G_0 T_p \\ T_n^x &= t_n^x + t_n^0 G_0 T_n^x + t_n^x G_0 T_n \end{aligned}$$

Faddeev Equation in the Diagram Representation: S. S. Kamalov *et al.*, Nucl. Phys. A **690**, 494 (2001)

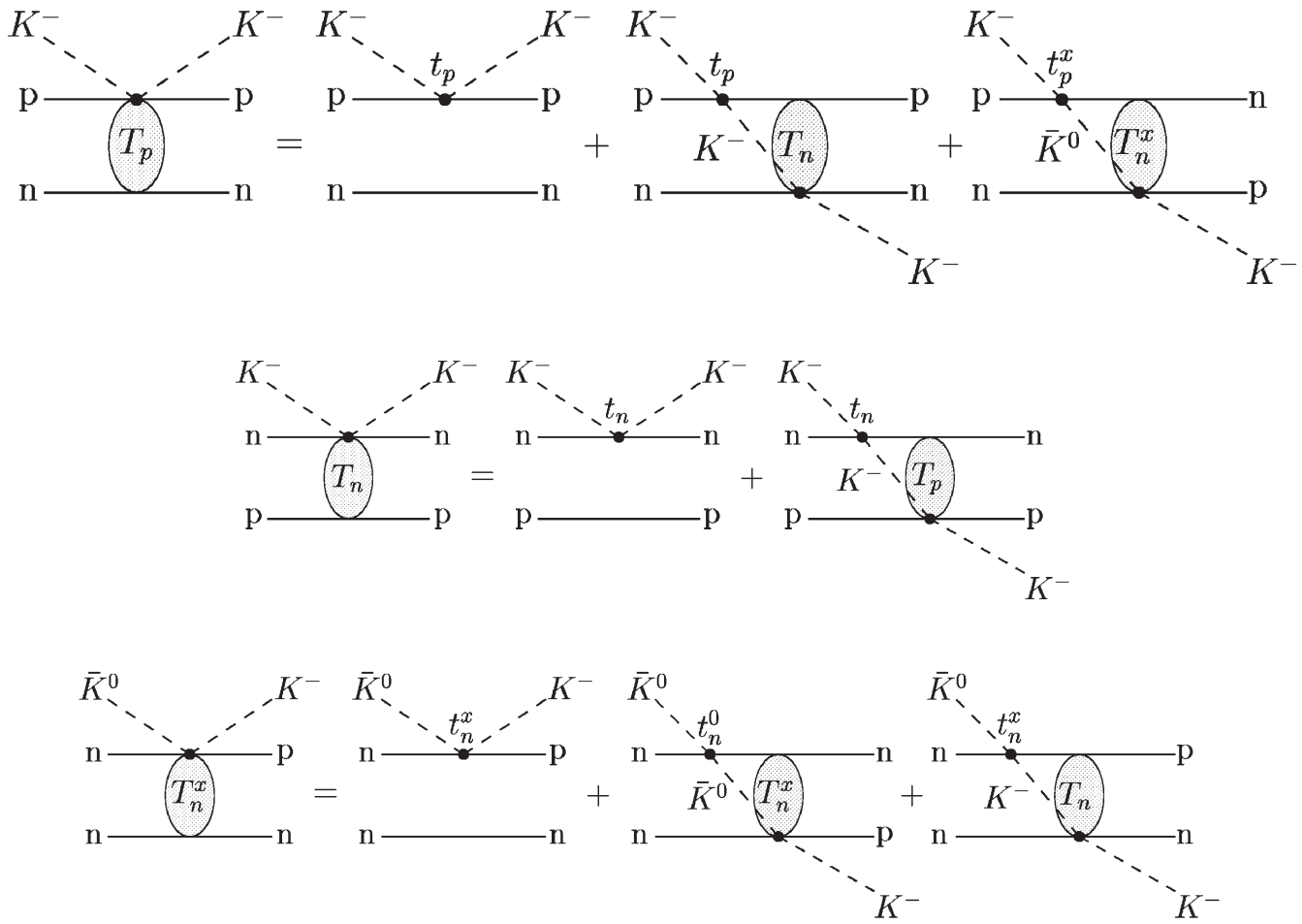


Figure: The Faddeev equation for the S–wave amplitude of low–energy $K^- d$ scattering in the diagram representation

Fixed centre approximation (FCA)

$$\hat{A}_{K^-d}(r) = \frac{\hat{a}_p + \hat{a}_n + \frac{2\hat{a}_p\hat{a}_n}{r} - \frac{\hat{a}_x^2}{r+\hat{a}_n^0} - \frac{2\hat{a}_x^2\hat{a}_n}{r(r+\hat{a}_n^0)}}{1 - \frac{\hat{a}_p\hat{a}_n}{r^2} + \frac{\hat{a}_x^2\hat{a}_n}{r^2(r+\hat{a}_n^0)}}.$$

$$\begin{aligned}
\hat{a}_p &= \left(1+\frac{m_K}{m_N}\right)\tilde{a}_{K^-p}(K^-p) \quad , \quad \hat{a}_n = \left(1+\frac{m_K}{m_N}\right)\tilde{a}_{K^-n}(K^-n), \\
\hat{a}_x &= \left(1+\frac{m_K}{m_N}\right)\tilde{a}_{K^-p}(\bar{K}^0n) \quad , \quad \hat{a}_n^0 = \left(1+\frac{m_K}{m_N}\right)\tilde{a}_{\bar{K}^0n}(\bar{K}^0n),
\end{aligned}$$

P–wave Scattering Length of Kaonic Deuterium

Energy level displacement of kaonic deuterium in $2p$ state

$$\epsilon_{np} = -\frac{2}{3}\frac{\alpha^5}{n^3}\left(1-\frac{1}{n^2}\right)\left(\frac{m_K m_d}{m_K+m_d}\right)^4 \mathrm{Re}\,\tilde{a}^{(1)}_{K^-d}(K^-d),$$

$$\Gamma_{np} = \frac{4}{9}\frac{\alpha^5}{n^3}\left(1-\frac{1}{n^2}\right)\left(\frac{m_K m_d}{m_K+m_d}\right)^4 \mathrm{Im}\,\tilde{a}^{(1)}_{K^-d}(K^-d)$$

Faddeev equation

$$\begin{aligned}
T_p^{(1)} &= t_p^{(1)} + t_p^{(1)}G_0T_n^{(0)} + t_p^{(0)}G_0T_n^{(1)} + t_p^{x(1)}G_0T_n^{x(0)} + t_p^{x(0)}G_0T_n^{x(1)}\,,\\
T_n^{(1)} &= t_n^{(1)} + t_n^{(1)}G_0T_p^{(0)} + t_n^{(0)}G_0T_p^{(1)}\,,\\
T_n^{x(1)} &= t_n^{x(1)} + t_n^{0(1)}G_0T_n^{x(0)} + t_n^{0(0)}G_0T_n^{x(1)} + t_n^{x(1)}G_0T_n^{(0)} + t_n^{x(0)}G_0T_n^{(1)}.
\end{aligned}$$

Solution of the Faddeev equation in the FCA

$$\tilde{a}^{(1)}_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3x \, |\Phi_d(\vec{r})|^2 \, \hat{A}^{(1)}_{K^-d}(r)$$

$$\hat{A}^{(1)}_{K^-d}(r) = \hat{A}^{(1)}_{p}(r) + \hat{A}^{(1)}_{n}(r)$$

Solution of the Faddeev equation in the FCA

$$\hat{A}^{(1)}_{K^-d}(r) = \hat{A}^{(1)}_{p}(r) + \hat{A}^{(1)}_{n}(r)$$

$$\hat{A}^{(1)}_{p}(r) = \frac{F_p(r) + \left[\frac{2}{3}\frac{a_p^{(0)}}{r} - \frac{\frac{4}{9}\left(\frac{a_x^{(0)}}{r}\right)^2}{1+\frac{2}{3}\frac{a_n^{0(0)}}{r}}\right] F_n}{1 - \frac{4}{9}\frac{a_p^{(0)}a_n^{(0)}}{r^2} - \frac{\frac{8}{27}\left(\frac{a_x^{(0)}}{r}\right)^2\frac{a_n^{(0)}}{r}}{1+\frac{2}{3}\frac{a_n^{0(0)}}{r}}}$$

$$\hat{A}^{(1)}_{n}(r) = F_n(r) + \frac{2}{3}\frac{\hat{a}_n^{(0)}}{r}\hat{A}^{(1)}_{p}(r)$$

$$F_p(r) = a_p^{(1)} + \frac{2}{3}\frac{a_p^{(1)}}{r}\hat{A}^{(0)}_{n}(r) - \frac{2}{3}\frac{a_x^{(1)}}{r}\hat{A}^{x(0)}_{n}(r)$$

$$F_n(r) = a_n^{(1)} + \frac{2}{3}\frac{a_n^{(1)}}{r}\hat{A}^{(0)}_{p}(r)$$

Theoretical Basis of our Researches

Chiral Lagrangians with $SU(3) \times SU(3)$ chiral symmetry

$$\mathcal{L}_{\rm int}[P(x), B(x), B(x), \ldots] =$$

$$= \langle \bar{B}(x) i\gamma^\mu [s_\mu(x), B(x)] \rangle$$

$$- g_A\,(1-\alpha_D)\,\langle \bar{B}_j(x)\gamma^\mu [p_\mu(x), B(x)]\rangle - g_A\,\alpha_D\,\langle \bar{B}(x)\gamma^\mu \{p_\mu(x), B(x)\}\rangle$$

$$+ g_{\Lambda^*}\,\bar{\Lambda}^*(x)\gamma^\mu\gamma^5\langle p_\mu(x) B(x)\rangle$$

$$+ \langle \bar{B}_j(x) i\gamma^\mu [s_\mu(x), B(x)]\rangle$$

$$- g_{A_j}\,(1-\alpha_{D_j})\,\langle \bar{B}_j(x)\gamma^\mu [p_\mu(x), B(x)]\rangle - g_{A_j}\,\alpha_{D_j}\,\langle \bar{B}_j(x)\gamma^\mu \{p_\mu(x), B(x)\}\rangle$$

$$\sqrt{2}\, g_A \bar{D}^{abc}_\mu(x)\Theta^{\mu\nu}\gamma^5 (p_\nu(x))^d_a B^e_b(x)\,\varepsilon_{cde}$$

$B(\underline{\mathbf{8}}) = (p, n, \Lambda^0, \Sigma, \Xi)$, $\Lambda^* = \Lambda^0(1405)(\underline{\mathbf{1}})$
$B_1(\underline{\mathbf{8}}) = (N(1440), \Lambda(1600), \Sigma(1660))$
$B_2(\underline{\mathbf{8}}) = (N(1710), \Lambda(1810), \Sigma(1880))$
$D(\underline{\mathbf{10}}) = (\Delta(1232), \Sigma^*(1385), \ldots)$

Experimental Basis of our Researches

Experimental ratios of cross sections for inelastic K^-p scattering at threshold

$$\gamma = \frac{\sigma(K^-p \to \Sigma^-\pi^+)}{\sigma(K^-p \to \Sigma^+\pi^-)} = 2.360 \pm 0.040$$

$$R_c = \frac{\sigma(K^-p \to \Sigma^-\pi^+) + \sigma(K^-p \to \Sigma^+\pi^-)}{\sum_{Y\pi}\sigma(K^-p \to Y\pi)} = 0.664 \pm 0.011$$

$$R_n = \frac{\sigma(K^-p \to \Lambda^0\pi^0)}{\sigma(K^-p \to \Sigma^0\pi^0) + \sigma(K^-p \to \Lambda^0\pi^0)} = 0.189 \pm 0.015.$$

Experimental data on S–wave K^-p scattering length by SMI

$$\tilde{a}^{DEAR}_{K^-p} = (-0.468 \pm 0.090 \pm 0.015) + i\,(0.302 \pm 0.135 \pm 0.036)\ \text{fm}$$